Supplementary Materials

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Theorem Suppose the difficulty, d, is fixed and all workers' γ are equal. Let X_t be a random variable denoting the number of workers who have provided the first-seen incorrect answer after t workers have provided incorrect answers. Then, $\theta < \frac{1-2(1-d)^\gamma}{(1-d)^\gamma}$ if and only if the expected probability the next worker returns the first-seen incorrect answer is greater than the probability the next worker returns the correct answer. In other words, for all t, $\theta < \frac{1-2(1-d)^\gamma}{(1-d)^\gamma}$ if and only if

$$(1-(1-d)^\gamma)\cdot E[\frac{X_t}{t+\theta}] > (1-d)^\gamma$$

Proof Let I_j be a random variable indicating whether or not worker number j sits at the table. Then, $X_t = I_1 + \ldots + I_t$. Define $P(I_1 = 1) = 1$.
By our generative model,

$$\begin{aligned} P(I_j = 1) &= \sum_{k=1}^{j-1} P(X_{j-1} = k)\frac{k}{j-1+\theta} \\ &= E[\frac{X_{j-1}}{j-1+\theta}] \end{aligned}$$

Thus,

$$\begin{aligned} E[X_t] &= (1 + E[\frac{X_1}{1+\theta}] + \ldots + E[\frac{X_{t-1}}{t-1+\theta}]) \\ &= (E[X_{t-1}] + E[\frac{X_{t-1}}{t-1+\theta}]) \\ &= E[X_{t-1}] + E[\frac{X_{t-1}}{t-1+\theta}] \\ &= E[X_{t-1}] + \frac{1}{t-1+\theta}\cdot E[X_{t-1}] \\ &= (1 + \frac{1}{t-1+\theta})E[X_{t-1}] \\ &= \frac{t+\theta}{t-1+\theta}E[X_{t-1}] \end{aligned}$$

We see then, that $E[X_t] = \frac{t+\theta}{1+\theta}$. Therefore, we have $E[\frac{X_t}{t+\theta}] = \frac{1}{\theta+1}$ for all t. Therefore,

$$\begin{aligned}
\theta &< \frac{1-2(1-d)^\gamma}{(1-d)^\gamma} \\
\theta &< \frac{1-(1-d)^\gamma}{(1-d)^\gamma} - \frac{(1-d)^\gamma}{(1-d)^\gamma} \\
\theta + 1 &< \frac{1-(1-d)^\gamma}{(1-d)^\gamma} \\
\frac{(1-d)^\gamma}{1-(1-d)^\gamma} &< \frac{1}{\theta+1} \\
\frac{(1-d)^\gamma}{1-(1-d)^\gamma} &< E[\frac{X_t}{t+\theta}] \\
(1-d)^\gamma &< (1-(1-d)^\gamma) \cdot E[\frac{X_t}{t+\theta}]
\end{aligned}$$